Parity violating target asymmetry in electron - proton scattering

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Abstract

We analyze the parity-violating (PV) components of the analyzing power \mathcal{A} in elastic electron-proton scattering and discuss their sensitivity to the strange quark contributions to the proton weak form factors. We point out that the component of \mathcal{A} along the momentum transfer is independent of the electric weak form factor and thus compares favorably with the PV beam asymmetry for a determination of the strangeness magnetic moment. We also show that the transverse component could be used for constraining the strangeness radius. Finally, we argue that a measurement of both components could give experimental information on the strangeness axial charge.

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1 Introduction

Parity violating (PV) electron scattering can provide very interesting information on the electroweak (ewk) structure of the nucleon, as first suggested in Ref.[1, 2], complementary to those given by neutrino scattering and by PV atomic experiments.

Only three of the four form factors determining the proton weak neutral currents (excluding second class currents) can be probed by the PV electron scattering. In fact, the induced pseudoscalar component of the axial-vector current does not contribute to PV electron scattering to leading order in ewk coupling.

In particular it could shed light on the possible strange quark contributions to the nucleon properties. Such a contribution is strongly suggested by the analysis of the pion-nucleon sigma term [3, 4, 5], of the cross section of the elastic neutrino/antineutrino-proton scattering [6, 7], and of the spin-dependent structure functions of the nucleon in deep inelastic scattering [8, 9, 10, 11].

A first measurement of the PV beam asymmetry in $\vec{e} - p$ elastic scattering was performed at Bates/MIT Laboratory by the SAMPLE Collaboration giving the first experimental determination of the weak magnetic form factor of the proton at Q²= 0.1 (GeV/c)² [12]. From this value the strangeness magnetic form factor is obtained by subtracting out the neutron and proton magnetic form factors as $G_M^s(0.1 \text{ (GeV/c})^2) = 0.23 \pm 0.37 \pm 0.15 \pm 0.19 \mu_N$, where the three uncertainties are statistical, systematic and theoretical, respectively.

This result suggests a positive value of the strangeness magnetic moment μ_s = $G_M^s(0)$, even if a vanishing or a negative value can not be excluded. This is at variance with the large majority of the theoretical predictions and could lead to rather bizarre properties of QCD [13].

A second measurement of the PV beam asymmetry at Q^2 =0.48 (GeV/c)² has been made very recently by the HAPPEX Collaboration [14] at the Thomas Jefferson Laboratory. Unlike the SAMPLE Collaboration, they have chosen to detect the scattered electrons at forward angles so as to provide information on the strange contribution to the weak electric form factor \tilde{G}_E . The measured asymmetry (\mathcal{A}_{LR} = - 14.5 ± 2.2 ppm) turns out to be explained by the ewk Standard Model without contributions from the strange quark.

Because of the difficulties inherent in the PV electron scattering experiment an independent determination of μ_s , as well as of the other strangeness properties of the proton (mean-square radius and axial charge), could be extremely useful.

The literature is already rich of several theoretical studies [2, 15, 16, 17, 18] and experimental proposals [19, 20, 21, 22] of PV electron scattering from complex nuclei. In the case of the deuteron, the PV helicity asymmetry of the exclusive electrodisintegration cross section has also been considered [23].

Another opportunity not yet explored is given by the measurement of the PV asymmetry of the analyzing power arising in the scattering of unpolarized electrons from polarized protons. In principle, the PV target asymmetry is even more versatile than the PV beam asymmetry for disentangling the different weak form factors because the polarization of the proton target can be freely chosen whereas the electron beam can be polarized only along the beam momentum.

In this paper we intend to study such target asymmetry in the low Q^2 range and to analyze its potential for an experimental determination of the ewk form factors.

The rapid and continuous improvements in the production of polarized targets [24] promise to make feasible in the near future such a measurement. In fact, experiments with polarized hydrogen targets are nowadays very common either using solid targets of hydrogenous compounds such as NH₃ or Butanol with external beams or gas targets with internal beams. High proton spin polarization ($\sim 90\%$) can be achieved in solid targets using the method of dynamic nuclear polarization together with the 3 He/ 4 He dilution refrigerator technique. Also the polarization reversal is now a relatively fast process with high spin flip efficiency. Further, the production of polarized internal targets in storage rings seems to be very promising because it allows one to have pure atomic species, high polarization and polarization reversibility on a time scale of msec. Moreover, high thickness can be reached thanks to the laser optical pumping, together with a free choice of the spin orientation using Helmholtz coils. In conclusion, a measurement of the PV analyzing power in the $e-\vec{p}$ scattering seems within the reach of the present experimental capability, provided the entire apparatus is kept stable for the long time required.

Note that other more complicated PV observables are possible in the electron proton elastic scattering if both beam and target are polarized. However, they are out of the experimental feasibility at the moment, and thus we shall not consider them in this paper.

Alternatively to the measurement of the PV asymmetry in the analyzing power, the PV polarization of the recoiling proton for unpolarized beam and target could be studied [25]. However, even if the recoil polarimetry has developed rapidly in recent years, the PV longitudinal and sideways components of the polarization vector have values which are typically of the order of 10^{-5} at low Q^2 , i.e. three-order of magnitude smaller than those measurable with the actual focal plane polarimeters.

A work on target asymmetry has been published some years ago by Fayyazuddin and Riazuddin [26] who limited their considerations to the longitudinal component of the analyzing power. However, their results are useful for the order of magnitude, at the most, because their analytic expression contains several errors (an overall factor of 2, a sign error and an erroneous dependence on the form factors in the eletromagnetic (em) - axial interference term). This is rather strange because the effects of the neutral currents in the elastic electron-proton scattering had already been considered by different authors in the 70's, a few years after the appearance of the unified gauge theory of weak and em interactions [27, 28]. While much work was concentrated on the polarized beam asymmetry, in [29, 30, 31] the case of the polarized target asymmetry was also considered. In these papers the analytic expression of the PV target asymmetry is incorrect only in the sign of the em axial interference term [29, 30] or in the sign of the em - vector interference term [31]. Note that these papers were mainly concerned with the discrimination among various SU(2)X U(1) models and other models based on larger gauge groups, such as $SU(2)_L \times SU(2)_R$ or $SU(3) \times U(1)$.

Finally, we have to mention that both PV beam and target asymmetries in the electron-proton scattering have also been investigated [32] in the GeV range to detect manifestations of a possible nonstandard weak boson Z^{0} . In this paper analytic formulas for the asymmetries are not reported because they are evaluated in a numerical way starting from the ewk currents.

The rest of the paper is organized as follows. In Sec.2 we describe our formulation of the elastic $e - \vec{p}$ scattering and we give the expressions of the target asymmetries. In Sec.3 we present and discuss our numerical results. Finally, in Sec.4 we state our conclusions.

2 Formalism

2.1 Parity-violating elastic cross section

The invariant amplitude for the parity-violating elastic electron proton scattering to lowest order, is the sum of the one-photon and the one- Z^0 boson exchange process:

$$\mathcal{M} = \mathcal{M}_{[\gamma]} + \mathcal{M}_{[Z]} \qquad , \tag{1}$$

with

$$\mathcal{M}_{[\gamma]} = -\frac{4\pi\alpha}{Q^2} j_{\mu} J^{[\gamma]\mu} \qquad , \tag{2}$$

and

$$\mathcal{M}_{[Z]} = \frac{G}{2\sqrt{2}} (g_V^e j_\mu + g_A^e j_{\mu 5}) J^{[NC]\mu} \qquad , \tag{3}$$

in the limit $Q^2 \ll M_Z^2$, which we are interested in; $Q^2 = -q_\mu^2 > 0$ is the four momentum transfer squared, α is the fine-structure constant, G is the weak Fermi constant. Finally, g_V^e and g_A^e are the neutral vector and axial-vector couplings of the electron which, in the Standard Model, are given by $g_A^e = 1$, $g_V^e = -1 + 4 \sin^2 \vartheta_W$, ϑ_W being the Weinberg or weak-mixing angle. The conventions of Musolf et al. [33] for the weak coupling constants are assumed.

The electron vector and axial-vector currents are given by the Dirac form

$$j_{\mu} = \bar{u}(k', s'_e) \gamma_{\mu} u(k, s_e) ,$$

$$j_{\mu 5} = \bar{u}(k', s'_e) \gamma_{\mu} \gamma_5 u(k, s_e) , \qquad (4)$$

where (k, s_e) and (k', s'_e) are the four-momentum and the covariant spin four-vector of the incoming and outgoing electron, respectively. We recall the properties $k_e \cdot s_e = k'_e \cdot s'_e = 0$ and $s_e^2 = s'_e^2 = -1$.

As for the nucleonic currents, $J_{\mu}^{[\gamma]}$ is the em current and $J_{\mu}^{[NC]}$ the neutral current which consists of a vector and an axial-vector component

$$J_{\mu}^{[NC]} = J_{\mu}^{[V]} + J_{\mu}^{[A]} \qquad . \tag{5}$$

Using the Gordon decomposition, the general expressions of the matrix elements of the nucleon ewk currents consistent with Lorentz covariance and with parity and time-reversal invariance can be written in the useful form

$$J_{\mu}^{[\gamma]} = \bar{u}(p',s') \left[G_M \gamma_{\mu} + \frac{(G_E - G_M)}{(1+\tau)} \frac{(p+p')_{\mu}}{2M} \right] u(p,s) ,$$

$$J_{\mu}^{[V]} = \bar{u}(p',s') \left[\tilde{G}_{M} \gamma_{\mu} + \frac{(\tilde{G}_{E} - \tilde{G}_{M})}{(1+\tau)} \frac{(p+p')_{\mu}}{2M} \right] u(p,s) , \qquad (6)$$

$$J_{\mu}^{[A]} = \bar{u}(p',s') \left[\tilde{G}_{A} \gamma_{\mu} + i(\tilde{G}_{P}/M) q_{\mu} \right] \gamma_{5} u(p,s) ,$$

where M is the nucleon mass, $\tau = Q^2/4M^2$. As before, the proton four-momenta and spin four-vectors satisfy the relations $p \cdot s = p' \cdot s' = 0$ and $s^2 = s'^2 = -1$.

In the following, we do not need to care about the induced pseudoscalar current which, being proportional to $\gamma_5 q_\mu$, does not contribute to observables in PV electron scattering to leading order in ewk coupling when saturated with the leptonic tensor.

Assuming, as usual, the extreme relativistic limit (ERL) for the electron ($m_e \ll E_e$) and neglecting the purely weak component term, which is $\sim G^2$, the elastic cross section in the case of an unpolarized electron beam and a polarized nucleon target, can be written

$$\frac{d\sigma}{d\Omega_{e'}} = \frac{\alpha^2}{Q^4} \left(\frac{E_{e'}}{E_e}\right)^2 \sum_{s'} \left\{ L^0_{\mu\nu} W^{\mu\nu}_{[\gamma]} - g_{eff} \left(g^e_V L^0_{\mu\nu} + g^e_A L^h_{\mu\nu}\right) \left(W^{\mu\nu}_{[\gamma V]} + W^{\mu\nu}_{[\gamma A]}\right) \right\} , \tag{7}$$

where we have averaged over the initial electron spin states and summed over the final ones. The effective weak coupling constant g_{eff} determining the magnitude of the PV effects in the low and medium Q^2 is given by

$$g_{eff} = \frac{Q^2}{4\pi\alpha} \frac{G}{2\sqrt{2}} \quad , \tag{8}$$

and

$$L^{0}_{\mu\nu} = 2\left(k_{\mu}k'_{\nu} + k_{\nu}k'_{\mu} - \frac{Q^{2}}{2}g_{\mu\nu}\right) ,$$

$$L^{h}_{\mu\nu} = 2 i \epsilon_{\mu\nu\alpha\beta}q^{\alpha}k^{\beta} , \qquad (9)$$

are the well known symmetric and antisymmetric leptonic tensors, where $\epsilon_{\mu\nu\alpha\beta}$ is the totally antisymmetric Levi-Civita tensor.

Exploiting Lorentz and CP invariance (but allowing for parity violation), the hadronic tensor can be expressed in the general form [34]

$$W_{[a]}^{\mu\nu} = F_1^{[a]}(Q^2)g^{\mu\nu} + F_2^{[a]}(Q^2)\frac{p^{\mu}p^{\nu}}{M^2}$$

$$+ i\epsilon^{\mu\nu\alpha\beta} \left[F_3^{[a]}(Q^2)\frac{p_{\alpha}q_{\beta}}{M^2} + g_1^{[a]}(Q^2)\frac{q_{\alpha}s_{\beta}}{M} + g_2^{[a]}(Q^2)\frac{p_{\alpha}s_{\beta}}{M} \right]$$

$$+ g_3^{[a]}(Q^2)\frac{p^{\mu}s^{\nu} + p^{\nu}s^{\mu}}{M} + g_4^{[a]}(Q^2)\frac{(q \cdot s)p^{\mu}p^{\nu}}{M^3} + g_5^{[a]}(Q^2)\frac{q \cdot s}{M}g^{\mu\nu} ,$$

$$(10)$$

where $a = \gamma, \gamma V, \gamma A$. In the case of the elastic scattering, the structure functions of the deep inelastic scattering case considered in [34], become the unpolarized form factors $F_i^{[a]}$ and the polarized ones $g_i^{[a]}$, while the structure of the tensor remains unchanged.

Note that in the literature are present other different (but equivalent) definitions of the hadronic tensor and of the structure functions obtained from each other by inserting or dropping terms proportional to q^{μ} or q^{ν} (which give no contribution in the ERL) and using suitable identities [34, 35, 36].

From the explicit calculation the non-zero form factors are

$$F_{1}^{[\gamma]} = -\tau G_{M}^{2} ,$$

$$F_{2}^{[\gamma]} = \frac{G_{E}^{2} + \tau G_{M}^{2}}{1 + \tau} ,$$

$$g_{1}^{[\gamma]} = -\frac{G_{M}(G_{E} + \tau G_{M})}{2(1 + \tau)} ,$$

$$g_{2}^{[\gamma]} = \tau \frac{G_{M}(G_{E} - G_{M})}{1 + \tau} ,$$

$$F_{1}^{[\gamma V]} = -2 \tau G_{M} \tilde{G}_{M} ,$$

$$F_{2}^{[\gamma V]} = 2 \frac{G_{E} \tilde{G}_{E} + \tau G_{M} \tilde{G}_{M}}{1 + \tau} ,$$

$$g_{1}^{[\gamma V]} = -\frac{G_{M} \tilde{G}_{E} + G_{E} \tilde{G}_{M} + 2\tau G_{M} \tilde{G}_{M}}{2(1 + \tau)} ,$$

$$g_{2}^{[\gamma V]} = \tau \frac{G_{M} \tilde{G}_{E} + G_{E} \tilde{G}_{M} - 2G_{M} \tilde{G}_{M}}{1 + \tau} ,$$

$$F_{3}^{[\gamma A]} = \tilde{G}_{A} G_{M} ,$$

$$g_{3}^{[\gamma A]} = \tilde{G}_{A} G_{E} ,$$

$$g_{4}^{[\gamma A]} = -\frac{\tilde{G}_{A} (G_{E} - G_{M})}{1 + \tau} ,$$

$$g_{5}^{[\gamma A]} = -\tilde{G}_{A} G_{M} .$$

Let us introduce the notation $\sigma(s) \equiv (d\sigma/d\Omega_{e'})$ to indicate the elastic cross section. As it is clear from the general structure of the hadronic tensor, Eq.(10), $\sigma(s)$ can be written

$$\sigma(s) = \sigma(0) + \Delta\sigma(s) \qquad , \tag{12}$$

where $\Delta \sigma(s)$ is the contribution deriving from the initial nucleon polarization as obtained from the corresponding part of the hadronic tensors $W_{[a]}^{\mu\nu}$.

From the symmetry properties in $\mu \leftrightarrow \nu$ of the leptonic and hadronic tensors, the

cross section, Eq.(7), reads

$$\sigma(0) = \frac{\alpha^2}{Q^4} \left(\frac{E_{e'}}{E_e}\right)^2 \left\{ L^0_{\mu\nu} W^{\mu\nu}_{[\gamma]}(S) - g_{eff} \left(g^e_A L^h_{\mu\nu} W^{\mu\nu}_{[\gamma A]}(A) + g^e_V L^0_{\mu\nu} W^{\mu\nu}_{[\gamma V]}(S)\right) \right\} , \qquad (13)$$

$$\Delta\sigma(s) = -\frac{\alpha^2}{Q^4} \left(\frac{E_{e'}}{E_e}\right)^2 g_{eff} \left(g_A^e L_{\mu\nu}^h W_{[\gamma V]}^{\mu\nu}(A) + g_V^e L_{\mu\nu}^0 W_{[\gamma A]}^{\mu\nu}(S)\right) , \qquad (14)$$

where $W_{[a]}^{\mu\nu}(S)$ and $W_{[a]}^{\mu\nu}(A)$ are the symmetric and antisymmetric components of the hadronic tensor. In the laboratory (lab) frame $(p^{\mu} = (M, \mathbf{0}))$ the cross section becomes

$$\sigma(0) = \sigma_M \frac{E_{e'}}{E_e} \frac{1}{\varepsilon(1+\tau)} \left\{ \left(\varepsilon G_E^2 + \tau G_M^2 \right) - 2 g_{eff} \left[g_V^e \left(\varepsilon G_E \tilde{G}_E + \tau G_M \tilde{G}_M \right) + g_A^e \sqrt{\tau(1+\tau)(1-\varepsilon^2)} G_M \tilde{G}_A \right] \right\} ,$$

$$(15)$$

$$\Delta\sigma(s) = -g_{eff} \, \sigma_M \frac{E_{e'}}{E_e} \, \frac{1}{\varepsilon(1+\tau)}$$

$$\times \left\{ g_A^e \left(\frac{1-\varepsilon}{M} \right) \left[(G_E \tilde{G}_M + \tilde{G}_E G_M)(k \cdot s) \right. \right.$$

$$- \left((G_E \tilde{G}_M + \tilde{G}_E G_M)(E_e + M) - 2G_M \tilde{G}_M (E_e - M\tau) \right) \frac{(q \cdot s)}{2M(1+\tau)} \right]$$

$$+ g_V^e \left(\frac{1}{2M^2 \tau} \right) \tilde{G}_A \left[2G_E (1-\varepsilon)(E_e - M\tau)(k \cdot s) \right.$$

$$+ \left. \left(2M\tau G_M - G_E \left(E_e (1-\varepsilon) + 2M\tau\varepsilon \right) \right) (q \cdot s) \right] \right\},$$

$$(16)$$

where

$$\sigma_M = \frac{\alpha^2 \cos^2(\vartheta_{e'}/2)}{4E_e^2 \sin^4(\vartheta_{e'}/2)} \qquad , \tag{17}$$

is the Mott cross section for elastic scattering of an electron from a structureless proton and the parameter ε , which gives the degree of transverse linear polarization of the virtual photon, is defined as

$$\varepsilon = \left[1 + 2(1+\tau)\tan^2(\vartheta_{e'}/2)\right]^{-1} \quad . \tag{18}$$

The cross section asymmetry due to the target polarization is defined as

$$A_s = \frac{\sigma(s) - \sigma(-s)}{\sigma(s) + \sigma(-s)} \qquad , \tag{19}$$

where $\sigma(\pm s)$ is the cross section when the initial proton is polarized along (s) or opposite (-s) an arbitrary direction $\hat{\mathbf{s}}$. Then, from Eq.(12), we see that $(\sigma(s) - \sigma(-s)) = 2 \Delta \sigma(s)$ and only the interference terms contribute to the asymmetry; moreover $(\sigma(s) + \sigma(-s)) = 2 \sigma(0) \simeq 2 \sigma_{[\gamma]}(0)$ because the contribution deriving from the em-weak interference can be disregarded.

As for the reference frame we take, as usual, the y-axis orthogonal to the scattering plane $(\hat{\mathbf{e}}_y // \mathbf{k} \times \mathbf{k}')$, while for the direction of the z-axis there are substantially two possible choices: z-axis along the direction \mathbf{k} of the incident beam or along the direction \mathbf{q} of the momentum transfer. In both cases, obviously, the only nonzero components of the asymmetry are those in the scattering plane, i.e. the transverse (\mathcal{A}_x) and the longitudinal (\mathcal{A}_z) ones.

From the theoretical point of view, the most convenient choice is to take the z-axis parallel to \mathbf{q} . In fact, this choice leads to longitudinal and transverse asymmetries which correspond to the virtual Compton scattering asymmetries \mathcal{A}_1 and \mathcal{A}_2 , respectively [34, 37].

We recall that the quantity actually measured in the deep inelastic electron proton scattering experiments with a polarized target, is the longitudinal asymmetry corresponding to the choice of the z-axis parallel to the beam momentum \mathbf{k} . In this case the correspondence with the virtual Compton scattering asymmetries is more complex and \mathcal{A}_z becomes a combination of \mathcal{A}_1 and \mathcal{A}_2 .

Assuming $\hat{\mathbf{e}}_z//\mathbf{q}$, we obtain for \mathcal{A}_x and \mathcal{A}_z

$$\mathcal{A}_{x} = \sqrt{2} g_{eff} \frac{g_{V}^{e} \sqrt{(1+\tau)\varepsilon(1+\varepsilon)} \tilde{G}_{A} G_{E} + g_{A}^{e} \left(G_{E} \tilde{G}_{M} + G_{M} \tilde{G}_{E}\right) \sqrt{\tau \varepsilon(1-\varepsilon)}}{\varepsilon G_{E}^{2} + \tau G_{M}^{2}} ,$$

(20)

$$\mathcal{A}_z = 2 g_{eff} \frac{g_V^e \sqrt{\tau(1+\tau)} \tilde{G}_A G_M + g_A^e G_M \tilde{G}_M \tau \sqrt{1-\varepsilon^2}}{\varepsilon G_E^2 + \tau G_M^2} .$$

Of course, the asymmetry components when the z-axis is parallel to \mathbf{k} are related to the previous ones by a simple rotation in the scattering plane. In fact, if $\vartheta_{\hat{kq}}$ is the angle between \mathbf{k} and \mathbf{q} , we have, with obvious notations

$$\mathcal{A}_{x_k} = \mathcal{A}_{x_q} \cos \vartheta_{\hat{kq}} - \mathcal{A}_{z_q} \sin \vartheta_{\hat{kq}} ,$$

$$\mathcal{A}_{z_k} = \mathcal{A}_{x_q} \sin \vartheta_{\hat{k}q} + \mathcal{A}_{z_q} \cos \vartheta_{\hat{k}q} \quad , \tag{21}$$

with

$$\sin \vartheta_{\hat{kq}} = \frac{M}{E_e} \sqrt{\frac{2 \varepsilon \tau}{1 - \varepsilon}} ,$$

$$\cos \vartheta_{\hat{kq}} = \left(1 + \frac{M}{E_e}\right) \sqrt{\frac{\tau}{1 + \tau}} .$$
(22)

Coming back to Eq.(20), several observations are in order. First, \mathcal{A}_z does not depend on \tilde{G}_E and then on the proton strangeness form factor $G_E^{(s)}$. This follows from the fact that only the transverse components of the neutral currents contribute to \mathcal{A}_z . Second, the dependence of \mathcal{A}_z on the proton axial form factor \tilde{G}_A is lowered with respect to that on the proton weak magnetic form factor \tilde{G}_M because $g_V^e \ll g_A^e$, in particular at backward angles where $\varepsilon \to 0$. This means that the longitudinal asymmetry \mathcal{A}_z may be an useful quantity for an experimental determination of the proton strange magnetic moment, allowing for a measurement of μ_s complementary to that coming from the helicity asymmetry in the reaction $\vec{ep} \to ep$ as in the SAMPLE experiment.

In principle, this independence of \tilde{G}_E makes more convenient a measurement of \tilde{G}_M in the target asymmetry than in the helicity asymmetry of the $\vec{e}p \to ep$ reaction, \mathcal{A}_{LR} , as it is clear recalling its expression [33]

$$\mathcal{A}_{LR} = -2 \ g_{eff} \ \frac{\left[g_A^e \left(\varepsilon \ G_E \tilde{G}_E + \tau G_M \tilde{G}_M\right) + g_V^e \sqrt{\tau (1+\tau)(1-\varepsilon^2)} G_M \tilde{G}_A\right]}{\varepsilon \ G_E^2 + \tau G_M^2} \ . \tag{23}$$

By the way, the longitudinal target asymmetry coincides, apart from the sign, with the helicity asymmetry for backward scattering ($\vartheta_{e'} = 180^{\circ} \Rightarrow \varepsilon = 0$) as a consequence of the helicity conservation in the ERL for the electrons.

A final remark following from the comparison of \mathcal{A}_{LR} with both the components of the target asymmetry, concerns the different sensitivity on \tilde{G}_A due to the different dependence on ε . In fact, unlike \mathcal{A}_{LR} , the terms containing the weak vector form factors in the target asymmetries, usually dominant because $g_A^e \gg g_V^e$, are suppressed at forward angles ($\varepsilon \to 1$) by the kinematic factors making substantial the impact of \tilde{G}_A .

Therefore, a determination of \tilde{G}_A alternative to that deriving from $\nu/\bar{\nu}$ scattering experiments could be carried out in this kind of PV electron scattering experiments.

However, as in actual experiments the scattering angle can not be less than $\sim 10^{\circ}$, the suppression mentioned above is only partial and thus a good knowledge of the weak vector form factors is a prerequisite of such an experiment.

2.2 Nucleon electromagnetic and weak form factors

From the structure of the em and weak vector current operators in terms of the SU(3)-singlet and -octet currents it follows that the nucleon weak vector form factors are given by

$$\tilde{G}_{E,M}(Q^2) = \frac{1}{2} \xi_V^{T=1} G_{E,M}^V(Q^2) \tau_3 + \frac{\sqrt{3}}{2} \xi_V^{T=0} G_{E,M}^S(Q^2)
+ \xi_V^{(0)} G_{E,M}^{(s)}(Q^2) ,$$
(24)

with $\tau_3=+1$, -1 for the proton and neutron, respectively. $G_{E,M}^{S(V)}$ is the isoscalar (isovector) combination of the em Sachs form factors, $G_{E,M}^{(s)}$ is the strange-quark contribution and the couplings are appropriate linear combinations of quark weak vector charges. In the Standard Model they have the values

$$\xi_V^{T=1} = 2(1 - 2\sin^2\theta_W)$$
 , $\sqrt{3}\xi_V^{T=0} = -4\sin^2\theta_W$, $\xi_V^{(0)} = -1$. (25)

The nucleon has not net strangeness, so that $G_E^{(s)}(0) = 0$. This is the only theoretical constraint about the strangeness form factors, which, according to [33], we take in the form

$$G_E^{(s)}(Q^2) = -\frac{1}{6} r_s^2 S^{[S]} Q^2 G_D^V(Q^2) (1 + \lambda_E^{(s)} \tau)^{-1} ,$$

$$G_M^{(s)}(Q^2) = \mu_s G_D^V(Q^2) (1 + \lambda_M^{(s)} \tau)^{-1} ,$$
(26)

i.e. an extension of the Galster parametrization [38] commonly used for the nucleon em form factors

$$G_E^p(Q^2) = G_D^V(Q^2) , \qquad G_E^n(Q^2) = -\mu_n \tau G_D^V(Q^2) (1 + 5.6\tau)^{-1}$$

$$G_M^p(Q^2) = \mu_p G_D^V(Q^2) , \qquad G_M^n(Q^2) = \mu_n G_D^V(Q^2) ,$$

$$(27)$$

where $G_D^V(Q^2)=(1+Q^2/M_V^2)^{-2}$, with a cut-off mass squared $M_V^2=0.71~GeV^2$.

The Sachs strangeness radius r_s^{2} [S], which characterizes the low Q^2 behaviour of $G_E^{(s)}$, is related to the more familiar Dirac strangeness radius

$$r_s^2 \equiv -6 \left[\frac{d \ F_1^{(s)}(Q^2)}{d \ Q^2} \right]_{Q^2=0}$$
 (28)

by the relation

$$r_s^{2[S]} = r_s^2 + \frac{3}{2M^2} \mu_s$$
 (29)

Very little is known about the values of μ_s and r_s^2 even if many calculations of the strangeness vector form factors have been carried out using different approaches (lattice calculations, effective Lagrangian, dispersion relations, hadronic models). The predictions of the strangeness moments are quite different in different approaches and can also largely vary within a given approach because of the need of additional assumptions and approximations. In particular, r_s^2 is predicted to be positive in the dispersion theory analysis of the nucleon isoscalar form factors [39, 40], of the same order of magnitude but negative by the chiral quark-soliton model [41] and negative but of two order of magnitude smaller by the kaon-loop calculations [42]. A negative value of r_s^2 is also preferred by the analysis [7] of the $\nu p/\bar{\nu}p$ elastic scattering data [6] which, however, has been criticized for the use of a unique cut-off mass for the three SU(3) axial-vector form factors.

The different existing models widely disagree also about sign and magnitude of μ_s which is predicted to range from $\mu_s = 0.4 \ \mu_N$ [43] in the chiral hyperbag model to $\mu_s = -0.75 \ \mu_N$ [44] using QCD equalities among the octet baryon magnetic moments.

Analogously to (24), the axial-vector form factor can be decomposed in terms of the 3rd and 8th SU(3) octet components and of the possible strange component

$$\tilde{G}_A(Q^2) = \frac{1}{2} \, \xi_A^{T=1} \, G_A^{(3)}(Q^2) \, \tau_3 + \frac{1}{2} \, \xi_A^{T=0} \, G_A^{(8)}(Q^2) + \xi_A^{(0)} \, G_A^{(s)}(Q^2) \quad , \tag{30}$$

with coupling constants dictated at the tree level by the quark axial charges

$$\xi_A^{T=1} = -2$$
 , $\xi_A^{T=0} = 0$, $\xi_A^{(0)} = 1$. (31)

Note that in this limit the isoscalar component of \tilde{G}_A fully comes from the strange quark contribution. Information on the $Q^2 = 0$ value of the SU(3) octet form factors derives from charged current weak interactions. From neutron β -decay

and strong isospin symmetry it follows $G_A^{(3)}(0) = (D+F) \equiv g_A = 1.2601 \pm 0.0025$ [45], while from hyperon β -decays and flavor SU(3) symmetry it follows $G_A^{(8)}(0) = (1/\sqrt{3})(3F-D) = 0.334 \pm 0.014$ [46], D and F being the associated SU(3) reduced matrix elements. The Q^2 dependence of these form factors can be adequately parametrized with a dipole form

$$G_D^A(Q^2) = (1 + Q^2/M_A^2)^{-2}$$
 , (32)

with a cut-off mass M_A =1.032 GeV. The same dipole form is suggested in [33] for the strange axial vector form factor

$$G_A^{(s)}(Q^2) = g_A^{(s)} G_D^A(Q^2) (1 + \lambda_A^{(s)} \tau)^{-1}$$
 (33)

Here again, lacking theoretical constraints on $G_A^{(s)}(0)$ and because of the model dependence of the theoretical estimates, values of $g_A^{(s)}$ have to be extracted from the experiments and the first indications came from the BNL $\nu p/\bar{\nu}p$ experiment [6] and from the EMC data [8].

As for the weak coupling constants we emphasize that the values (25) and (31) are those predicted by the ewk Standard Model at the tree-level. In a realistic evaluation of the amplitude of any electron-hadron process one has to consider the radiative corrections to these values. Such corrections $R_{V,A}$, amounting to a factor $(1+R_{V,A})$ in all the coupling constants except in $\xi_A^{T=0}$ which becomes $\sqrt{3}R_A^{T=0}$, are very difficult to calculate because they receive contributions from a variety of processes (higher-order terms in ewk theory, hadronic physics effects,...). They have been estimated by various authors (for a review see Ref.[33] and citations therein) using different approaches and approximations with results in qualitative agreement. More precisely, R_V are estimated to be of the order of a few percent and R_A of the order of some tenth of percent. Therefore, while R_V can be neglected, the radiative corrections R_A must, in principle, be taken into account.

3 Results

In this paper we report our results on the polarized target asymmetry in the momentum transfer region $Q^2 \leq 1$ (GeV/c²). As for the parameters which determine the Q^2 fall-off of the expressions (26,33) of the strange form factors, which are completely unknown, we take the values $\lambda_E^{(s)} = 5.6$, $\lambda_M^{(s)} = 0$, $\lambda_A^{(s)} = 0$. While the

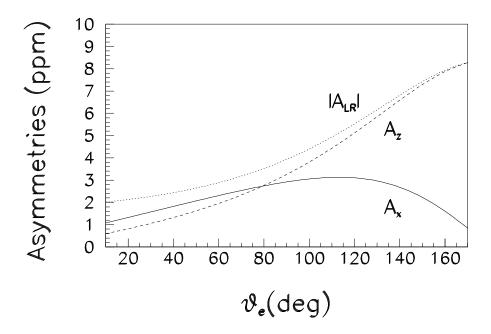


Figure 1: Angular distribution of the target asymmetry $\mathcal{A}_x(\vartheta_{e'})$ (full line), $\mathcal{A}_z(\vartheta_{e'})$ (dashed line) and of the modulus of the helicity asymmetry $\mathcal{A}_{LR}(\vartheta_{e'})$ (dotted line) at $Q^2=0.1(GeV/c)^2$, with $\mu_s=0.23~\mu_N$ [12] and $r_s^2=0.16~\mathrm{fm}^2$ [39].

value for $\lambda_E^{(s)}$ is suggested by the analogy with the Galster parametrization of G_E^n , the choice for $\lambda_M^{(s)}$ and $\lambda_A^{(s)}$ seems to us rather reasonable as we do not consider too high Q^2 values.

Furthermore, as reference values for the axial radiative corrections we adopt $R_A^{T=0} = -0.62$ and $R_A^{T=1} = -0.34$ given by Musolf and Holstein [48] using for the hadronic contributions the so-called best estimates for the weak meson-nucleon vertices of Ref.[47]. On the contrary, lacking a reliable estimate of the radiative correction to the strangeness axial coupling constant, we take $R_A^{(0)} = 0$. Finally, we use $g_A^{(s)} = -0.15$ as reference value of the strangeness axial charge, as deduced from the neutrino scattering experiment [6].

Let us start considering the angular distribution of the polarized target asymmetries. In Fig.1 we plot $\mathcal{A}_x(\vartheta_{e'})$ and $\mathcal{A}_z(\vartheta_{e'})$ for $Q^2=0.1$ (GeV/c)², calculated with Jaffe's value [39] of the strangeness radius ($r_s^2=0.16$ fm²) and with the central experimental value of the strange magnetic moment ($\mu_s=0.23 \mu_N$) [12]. The asymmetries, which are quite small at forward angles, show a remarkably different

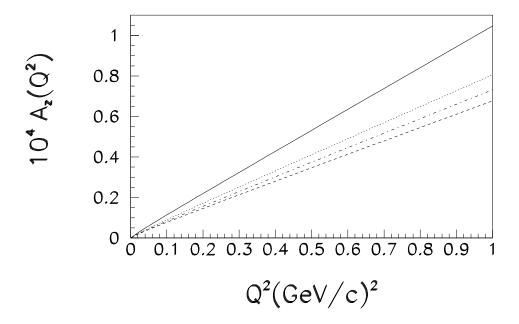


Figure 2: Dependence of the target asymmetry $\mathcal{A}_z(Q^2)$ on the strange magnetic moment μ_s , at $\vartheta_{e'}=170^\circ$ and for $r_s^2=0.16~\text{fm}^2~[39]$. The solid line is for $\mu_s=0.75~\mu_N~[44]$, the dashed line for $\mu_s=0.40~\mu_N~[40]$, the dotted line for $\mu_s=0$ and the dot-dashed line for $\mu_s=0.23~\mu_N~[12]$.

angular dependence as $\vartheta_{e'}$ increases. In fact, while $\mathcal{A}_x(\vartheta_{e'})$ increases very slowly, reaches a maximum for $\vartheta_{e'} \simeq 110^{\circ}$ and then decreases at backward angles, $\mathcal{A}_z(\vartheta_{e'})$ increases continuously with $\vartheta_{e'}$ reaching at backward angles values which are one order of magnitude greater than those at forward angles.

The angular behavior of the target asymmetries can be easily understood recalling their expression (20). In fact, at forward angles $\varepsilon \simeq 1$ and the surviving term in both the asymmetries is that proportional to g_V^e . On the contrary, at backward angles $\varepsilon \to 0$ and $\mathcal{A}_x(\vartheta_{e'}) \to 0$ while $\mathcal{A}_z(\vartheta_{e'})$ reaches its maximum.

For comparison, we have also drawn in Fig.1 the modulus of the helicity asymmetry $\mathcal{A}_{LR}(\vartheta_{e'})$ which has an angular dependence very similar to that of $\mathcal{A}_z(\vartheta_{e'})$. In fact, $|\mathcal{A}_{LR}|$ which is slightly greater than \mathcal{A}_z at forward angles, converges to \mathcal{A}_z for $\vartheta_{e'} \to 180^{\circ}$, as discussed in the previous section. There we pointed out that the longitudinal target asymmetry at backward angles may be an useful quantity for an experimental determination of μ_s . In order to show the effect on \mathcal{A}_z of variations in

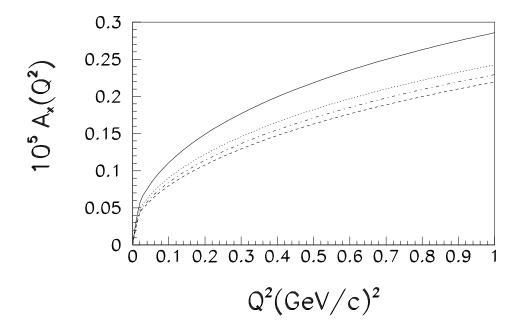


Figure 3: The same as in Fig.2 for the target asymmetry $\mathcal{A}_x(Q^2)$.

the strangeness magnetic moment we report in Fig.2 our results at $\vartheta_{e'}=170^{\circ}$ for a selected set of predictions of μ_s . Among the values given by the different models we have chosen those defining the theoretical range of μ_s , i.e. $\mu_s=-0.75~\mu_N$ [44] and $\mu_s=0.4~\mu_N$ [43]. Also reported are the curves corresponding to the experimental value of μ_s [12] and to $\mu_s=0$. Note that the Dirac strangeness radius has been held fixed, $r_s^2=0.16~\mathrm{fm^2}$ as deduced by Jaffe. This comparison makes evident the sensitivity on μ_s of the target asymmetry for electrons scattered in the backward direction, sensitivity which is very strong for $Q^2=1~\mathrm{(GeV/c)^2}$ but already noticeable even for lower Q^2 values.

Of course, because at backward angles A_z and $|A_{LR}|$ are nearly coincident, we can repeat for A_z all the considerations made for A_{LR} on the precision reachable in an extraction of μ_s from a measurement of A_{LR} . For example, at $Q^2=0.1$ (GeV/c)², the uncertainties affecting the other quantities determining the asymmetries (radiative corrections to the coupling constants, uncertainties in \tilde{G}_A arising from $g_A^{(s)}$ and M_A , ...) make it necessary a measurement within a 10% error to extract μ_s with a limit $|\delta\mu_s|\approx 0.22$ [33]. In the previous section we pointed out that the longitudinal target asymmetry has the important characteristic of being independent of \tilde{G}_E and then,

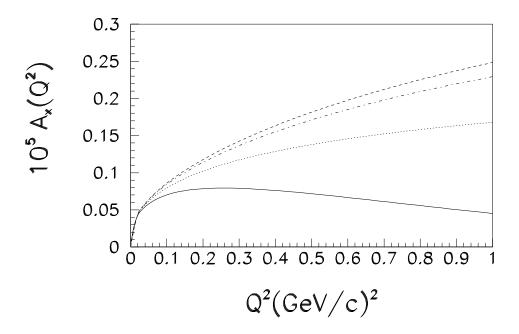


Figure 4: Dependence of the target asymmetry $\mathcal{A}_x(Q^2)$ on the strangeness radius r_s^2 , at $\vartheta_{e'}=170^\circ$ and for $\mu_s{=}0.23~\mu_N~$ [12]. The solid line is for $r_s^2{=}-0.32~$ fm $^2~$ [41], the dashed line for $r_s^2{=}0.21~$ fm $^2~$ [40], the dotted line for $r_s^2{=}0.16~$ fm $^2~$ [39].

in particular, of $G_E^{(s)}$. Unfortunately, the positive consequences of this fact, are made less significant by all the above mentioned uncertainties on the other parameters.

For completeness, we report in Fig.3 the transverse target asymmetry \mathcal{A}_x , in the same kinematical conditions as before and for the same set of values of μ_s . Even if \mathcal{A}_x shows a good sensitivity to μ_s , it is nearly an order of magnitude smaller than \mathcal{A}_z and then less suitable for an experimental measurement.

On the contrary, \mathcal{A}_x could be an useful quantity to constrain r_s^2 . According to Eq.(20), in order to minimize the impact of the poorly known \tilde{G}_A , it is convenient to consider backward scattered electrons. The sensitivity of \mathcal{A}_x to the strangeness radius is shown in Fig.4, where we compare our results of \mathcal{A}_x for $\vartheta_{e'} = 170^\circ$, having fixed the proton strangeness magnetic moment to its experimental value and for a restricted selection of predicted r_s^2 . Besides that given by Jaffe and $r_s^2 = 0$, we have used the two almost opposite values $r_s^2 = 0.21 \text{ fm}^2$, deduced by Hammer et al. [40] in their revised dispersion analysis and $r_s^2 = -0.32 \text{ fm}^2$ obtained by Kim et al.

[41] in a chiral quark soliton model.

At first sight, a measurement of \mathcal{A}_x in the backward direction could lead to discriminate between the different models. Actually, the precision on the extraction of r_s^2 from such experiment is strongly limited by the error induced by the uncertainty in the other quantities determining \mathcal{A}_x and particularly in μ_s . To be more quantitative on the precision reachable in a determination of r_s^2 , let us consider again the expression of \mathcal{A}_x , Eq.(20), and write it in the form

$$\mathcal{A}_x = \mathcal{A}_x^0 \left(1 + a_x \ r_s^2 + b_x \ \mu_s + c_x \ R_A^{T=1} + d_x \ R_A^{T=0} + e_x \ g_A^{(s)} \right) \quad , \tag{34}$$

which exibits the dependence on the strangeness radius, magnetic moment, axial charge and on the radiative corrections to the axial-vector coupling constants. The fractional change induced in the transverse asymmetry is given by

$$\frac{\delta \mathcal{A}_x}{\mathcal{A}_x} \simeq \left(a_x \ \delta r_s^2 + b_x \ \delta \mu_s + c_x \ \delta R_A^{T=1} + d_x \ \delta R_A^{T=0} + e_x \ \delta g_A^{(s)} \right) \quad . \tag{35}$$

The values of the coefficients in Eq.(34) and (35) for $\vartheta_{e'}=170^\circ$ and $Q^2=1(GeV/c)^2$ are $a_x=0.208~{\rm fm^{-2}}$, $b_x=-0.312$, $c_x=0.172$, $d_x=-0.048$, $e_x=-0.136$, and it turns out that the impact of $\delta\mu_s$, $\delta R_A^{T=1}$ and $\delta g_A^{(s)}$ on a determination of r_s^2 is strong. In fact, even with an experimental error of 10% in the measurement of \mathcal{A}_x , assuming $\delta R_A^{T=1}=\pm 0.28$ as estimated by Musolf and Holstein [48], $\delta g_A^{(s)}=\pm 0.09$ as given by neutrino scattering experiments [6], and assuming to know μ_s within ± 0.1 , one can not get for r_s^2 a precision better than $|\delta r_s^2|\approx 0.6$. It is to be concluded that the transverse asymmetry compares only slightly favorably with the beam helicity asymmetry as for the determination of r_s^2 .

Finally, we consider the possibilities provided by a measurement of target asymmetries for a determination of $g_A^{(s)}$. In Fig.5 we plot \mathcal{A}_x and \mathcal{A}_z at forward angle $(\vartheta_{e'}=10^\circ)$ as a function of Q^2 and for different values of $g_A^{(s)}$ within the range defined by neutrino experiments [6]. The other parameters are taken at their reference values. The variations induced by $g_A^{(s)}$ are very similar in \mathcal{A}_x and \mathcal{A}_z , slowly increasing with Q^2 and reach about 10% at $Q^2=1$ (GeV/c)². To give a quantitative estimate of the error with which $g_A^{(s)}$ could be extracted from a PV target asymmetry measurement, we limit our consideration to \mathcal{A}_z which does not take contribution from \tilde{G}_E . Therefore, analogously to Eq.(35), we have

$$\frac{\delta \mathcal{A}_z}{\mathcal{A}_z} \simeq \left(b_z \ \delta \mu_s + c_z \ \delta R_A^{T=1} + d_z \ \delta R_A^{T=0} + e_z \ \delta g_A^{(s)} \right) \quad , \tag{36}$$

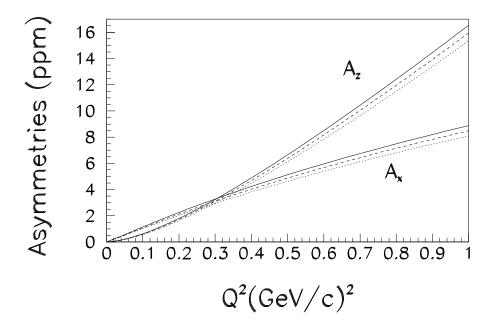


Figure 5: Dependence of the target asymmetry $\mathcal{A}_x(Q^2)$ and $\mathcal{A}_z(Q^2)$ on the axial charge $g_A^{(s)}$, at $\vartheta_{e'}=10^\circ$, for $\mu_s{=}0.23~\mu_N~$ [12] and $r_s^2{=}0.16~\mathrm{fm}^2~$ [39]. The dashed line is for the central value $g_A^{(s)}{=}{-}0.15$ while the full line and dotted line are for $g_A^{(s)}{=}{-}0.24$ and $g_A^{(s)}{=}{-}0.06$, respectively.

and from the evaluation at $Q^2=1$ (GeV/c)² and $\vartheta_{e'}=10^\circ$, we obtain $b_z=-0.243$, $c_z=0.473$, $d_z=-0.133$, $e_z=-0.375$.

Clearly, the largest effects on \mathcal{A}_z come from the uncertainties in $R_A^{T=1}$ and $g_A^{(s)}$, but also μ_s and $R_A^{T=0}$ have a sizeable impact. Therefore, a good knowledge of the radiative corrections and a precise determination of μ_s are requested in order that a measurement of \mathcal{A}_z could be exploited for constraining the value of $g_A^{(s)}$.

4 Conclusions

The aim of this paper was to extend the possible PV observables which could be used for an experimental determination of the weak form factors of the proton. To this end we have considered the asymmetries of the elastic electron-proton scattering cross section arising from the polarization of the proton target.

Concerning such an experiment, it seems to us that it should become feasible

in the near future because the tecnique of polarized target production has so much improved that it is nowadays possible to obtain hydrogen targets highly polarized, with an arbitrary direction of polarization, and with fast and efficient spin reversal.

First of all, we have carefully rederived the expression of the cross section in the electroweak theory valid for an arbitrary target polarization. Indeed, such a process was already addressed in the past by several authors but with incorrect results.

Then, we have decomposed the PV analyzing power, which is a vector lying in the scattering plane, considering the proton polarization along and perpendicular to the momentum transfer \mathbf{q} . This is indeed the most convenient decomposition from the theoretical point of view, leading to a longitudinal asymmetry independent of the electric weak form factor. For comparison, we recall that the helicity asymmetry of the $\vec{e} - p$ scattering depends also on \tilde{G}_E .

However, the actual calculations of A_z for backward angles, which is the most suitable kinematical situation for an experimental determination of μ_s , lead to an expected precision on μ_s which is of the same order as the one obtained in the helicity asymmetry experiment.

We have also shown that the transverse target asymmetry \mathcal{A}_x is rather sensitive to the strangeness radius r_s^2 in the case of backward detected electrons. However, it turns out from the calculations that the precision in the extraction of r_s^2 is strongly limited by the uncertainties in the other quantities (particularly in μ_s) affecting \mathcal{A}_x .

Another peculiarity of the target asymmetries A_x and A_z with respect to the beam asymmetry, is that their dependence on the axial form factor can be enhanced over that on the vector form factors. In fact, in the strict forward scattering ($\vartheta_{e'} = 0^{\circ}$) with the electron kinematical parameter $\varepsilon=1$, the target asymmetries are determined by \tilde{G}_A only. In practice, the finite dimensions of the detectors prevent too small scattering angles so that the influence of \tilde{G}_E and \tilde{G}_M can be greatly reduced but non completely cancelled. Anyway, a measurement of the target asymmetries in this kinematics could be useful for independently constraining the value of $G_A^{(s)}(Q^2)$ given by the neutrino scattering experiments.

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